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THE EFFECTS OF WALL TEMPERATURE AND SUCTION ON LAMINAR BOUNDARY-LAYER STABILITY

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William S. King

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Rand
SANTA MONICA, CA. 90406

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An integral solution of the two-dimensional boundary-layer equations for water with pressure gradient, heat transfer, and suction was employed to investigate laminar boundary-layer stability. It was shown that the effects of suction, wall heating, and pressure gradient on critical Reynolds numbers could be correlated as a function of universal boundary-layer parameters. It was indicated that suction is the most effective and pressure gradient the least effective means to stabilize a boundary layer. However, the effectiveness of suction is enhanced by a favorable pressure gradient. (Author)

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PREFACE

The research discussed in this report was sponsored by the Defense Advanced Research Projects Agency and is intended to provide a simple analytical means of estimating the effects of wall boundary conditions on boundary-layer stability. Accuracy was not the primary concern of the study. However, emphasis was placed on isolating the critical boundary-layer parameters and providing some quantitative prediction of their effects on boundary-layer stability.

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SUMMARY

An integral solution of the two-dimensional boundary-layer equations for water with pressure gradient, heat transfer, and suction was used to investigate laminar boundary-layer stability. There are theoretical bases to indicate that the effects of suction, wall heating, and pressure gradient on critical Reynolds numbers could be correlated as a function of universal boundary-layer parameters. Either the boundary-layer shape factor, H, or the parameter proposed by Wazzan, $\bar{\bar{u}}''(0)$, could be used. These two parameters are not independent. The preference of one parameter over the other depends on the nature of the data to be correlated. There is apparently no preference for theoretical data, and H is preferable for experimental data because the displacement and momentum thicknesses are easier to deduce from measurements than the wall-shear parameters.

The results of the analysis are used for a qualitative discussion of the effects of suction, wall heating, and pressure gradient on boundary-layer stability. Suction is the most effective, and pressure gradient the least effective, means to stabilize a boundary layer. Further, the stabilizing effects of suction and wall heating are enhanced by a favorable pressure gradient. The application of this analysis to calculations of the critical Reynolds number for an arbitrary body shape is discussed.

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CONTENTS

SYMBOLS

a = velocity profile parameter, Eq. (10)

b = temperature profile parameter, Eq. (11)

c = wave speed

c_p = specific heat

H = boundary-layer shape factor, Eq. (7)

P = pressure

Pr = Prandtl number

Q = suction parameter, Eq. (12)

R_c = critical Reynolds number

T = temperature

u = tangential velocity in the boundary layer

U_e = tangential velocity external to the boundary layer

v = normal velocity in the boundary layer

v_w = wall suction velocity

x = tangential coordinate

y = normal coordinate

α = wave number

δ = momentum layer thickness

δ^* = displacement thickness, Eq. (7)

δ_τ = thermal layer thickness

Δ = $\delta\tau/\delta$

ε = variable viscosity parameter, Eq. (12)

μ = molecular viscosity

λ = Pohlhausen, shape factor, Eq. (12)

κ = thermal conductivity

ρ = density

η = boundary layer coordinate

θ = momentum thickness, Eq. (7)

Θ = energy thickness, Eq. (7)

ϕ = variable viscosity parameter, Eq. (12)

Subscripts

e = external to the boundary layer

t = thermal layer

w = wall

x = derivative with respect to tangential coordinate

y = derivative with respect to normal coordinate

I. INTRODUCTION

In the study of boundary-layer control, it is well known that suction can completely stabilize a laminar boundary.[1] It is also known that heating is stabilizing for a laminar water boundary layer.[2,3] Both of these effects have been studied numerically. But as with most numerical studies, the interplay among critical parameters is not illuminated. The expense of conducting either a numerical or a physical experiment deters wide coverage of the various parameters that influence boundary-layer transition.

It is the purpose of this study to present a simple integral solution of self-similar flows that displays the relationship among the various boundary-layer profile parameters. These results will show that the critical Reynolds number as a function of pressure gradient, wall temperature, and wall suction can be correlated with a single boundary-layer parameter. The assumption of local similarity will be used in a discussion of the calculation of the critical Reynolds number for an arbitrary body shape.

II. ANALYSIS

BOUNDARY-LAYER FLOW

In this section, I shall develop the equations that define the boundary-layer profile parameters using an integral analysis. The boundary-layer stability parameters will be determined from the approximate Schlichting-Ulrich correlation and the Lin equations and the profiles derived in this section.

Consider the flow of water around a heated two-dimensional body. The two-dimensional laminar boundary-layer equations for an incompressible, variable-viscosity flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 , \tag{1}$$

$$\rho u u_x + \rho v u_y = - \frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) , \tag{2}$$

$$\rho c_p \left(u T_x + v T_y\right) = \kappa \frac{\partial^2 T}{\partial y^2} , \tag{3}$$

where u and v are the tangential and normal velocities, x and y are the coordinates tangential and normal to the body, the subscripts x and y denote the derivatives with respect to x and y, and μ is the variable viscosity that depends on the temperature. The specific heat, c_p, the density, ρ, and the thermal conductivity, κ, are assumed constant for heated water. The appropriate boundary conditions are:

$$y = \delta: \quad u = U_e, \quad u_y = u_{yy} = 0, \quad T = T_e, \quad T_y = T_{yy} = 0 ;$$

$$y = 0: \quad u = 0, \quad T = T_w, \quad \rho v_w u_y = \rho U_e \frac{dU_e}{dx} + \frac{d\mu}{dT}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y} + \mu_w \frac{\partial^2 u}{\partial y^2} ;$$

$$\rho v_w \frac{\partial T}{\partial y} = \kappa \frac{\partial^2 T}{\partial y^2} , \tag{4}$$

where v_w denotes the wall suction velocity.

The integral equations are obtained by integrating Eqs. (2) and (3) across the boundary layer. The results are:

$$\frac{d\theta}{dx} + \frac{U_{ex}}{U_e}\theta(2 + H) = \frac{\mu_w}{\rho U_e^2}\frac{\partial u}{\partial y_w} + \frac{v_w}{U_e} , \qquad (5)$$

$$\frac{d\Theta}{dx} = T_e v_w + \frac{\kappa}{\rho c_p}\frac{\partial T}{\partial y_w} , \qquad (6)$$

where

$$\theta = \int_0^\infty \frac{u}{U_e}\left(1 - \frac{u}{U_e}\right) dy ,$$

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U_e}\right) dy , \qquad (7)$$

$$H = \frac{\delta^*}{\theta} ,$$

$$\Theta = \int_0^\infty u(T_e - T)\, dy ,$$

I assume that the velocity and temperature profiles can be represented by fourth-order polynomials as expressed by the following:

$$\frac{u}{U_e} = a(\eta - 3\eta^2 + 3\eta^3 - \eta^4) + 6\eta^2 - 8\eta^3 + 3\eta^4 , \qquad (8)$$

$$\frac{T - T_w}{T_e - T_w} = b(\eta_t - 3\eta_t^2 + 3\eta_t^3 - \eta_t^4) + 6\eta_t^2 - 8\eta_t^3 + 3\eta_t^4 , \tag{9}$$

where $\eta = y/\delta$ and $\eta_t = y/\delta_\tau$. The profile parameters are defined by

$$a = \frac{\lambda + 12}{6 - \phi + Q} , \tag{10}$$

$$b = \frac{12}{6 + Q \frac{\Delta \, Pr}{\delta}} , \tag{11}$$

where

$$Q = \frac{\rho v_w}{\mu_w} ; \quad \phi = \left(\frac{d\mu}{dT}\right)_w \left(\frac{T_e - T_w}{\mu_w}\right) \frac{\delta b}{\delta_\tau} = \frac{\epsilon b}{\Delta} ,$$
$$\lambda = \frac{\rho}{\mu_w} \frac{dU_e}{dx} \delta^2 ; \quad \Delta = \frac{\delta_\tau}{\delta} . \tag{12}$$

For wall heating and a heated water boundary layer $\phi > 0$, and for suction $Q < 0$. Therefore, an increase in either or both of these parameters produces an increase in a.

So far the analytical procedure is similar to that used by Hauptmann,[3] except that here emphasis is also being placed on the effects of pressure gradient and suction on boundary-layer stability.

If the profiles given by Eqs. (8) and (9) are used to evaluate the parameters in the boundary-layer equation, one finds that

$$\frac{\theta}{\delta} = \frac{4}{35} + \frac{a}{(5)\ (21)} - \frac{a^2}{(36)\ (7)} , \tag{13}$$

$$\frac{\delta^*}{\delta} = \frac{3 - a}{20} , \tag{14}$$

$$\frac{\Theta}{\delta_\tau} = \left[\frac{4}{35} + \frac{a}{20} - \frac{(a+b)17}{(35)\ (24)} - \frac{ab}{(35)\ (7)}\right] U_e(T_e - T_w) \qquad Pr \sim 1 \ , \qquad (15a)$$

$$\frac{\Theta}{\delta_\tau} \approx \Delta \frac{a}{60}\ (6 - b)\ U_e(T_e - T_w) \qquad PR >> 1 \ ; \qquad \Delta << 1 \ . \qquad (15b)$$

Equations (15a) and (15b) represent the appropriate cases for a heated water boundary layer. Substituting Eqs. (8) and (9) into Eqs. (5) and (6)

$$\frac{d\theta}{dx} + \frac{U_{ex}}{U_e}\,\theta(2 + H) = \frac{\mu_w}{\rho U_e \delta}\,(a + Q) \ , \qquad (16)$$

$$\frac{d\Theta}{dx} = \frac{(T_e - T_w)}{\delta_\tau}\,\frac{\kappa}{\rho c_p}\,[b + \Delta QPr] \ . \qquad (17)$$

For the self-similar Falkner-Skan flows, a, b, Q, $\frac{\delta_\tau}{\delta}$, and ϕ are constants, and $U_e \sim x^m$. The wall temperature is a constant, and the slight variations in T_e will be neglected.

Solving Eqs. (16) and (17) for δ and Δ yields

$$\delta^2 = \frac{2x\mu_w(a + Q)}{\rho U_e(1 + 4m + 2mH)\left(\frac{4}{35} + \frac{a}{(5)\ (21)} - \frac{a^2}{(36)\ (7)}\right)} \qquad (18)$$

and

$$\Delta^2 = \frac{2x\mu_w\{b + \Delta QPr\}}{\delta U_e\ \delta^2 Pr\left[\frac{4}{35} + \frac{a}{20} + \frac{(a+b)17}{(35)\ (24)} - \frac{ab}{(36)\ (7)}\right]} \qquad Pr \cong 1 \qquad (19a)$$

$$\Delta^3 \cong \frac{2\mu_w x}{\rho U_e \delta^2}\,Pr[b + \Delta QPr] \qquad Pr >> 1 \ . \qquad (19b)$$

These equations are not explicit expressions for δ^2 and Δ because the boundary-layer parameters a, b, and Q contain either δ or δ_τ or both. Of course, they could be solved numerically. However, to find the important parameters I shall use simplifying assumptions of local similarity.

With flat-plate local similarity,

$$\delta_0^2 = \frac{4x\mu_w}{\rho U_e}\frac{315}{37}$$

$$\Delta_0 = Pr^{-1/2} \qquad Pr \sim 1$$

$$\Delta_0 = Pr^{-1/3} \qquad PR >> 1 \ . \tag{20}$$

Using the above values in the equations for a, b, Q, and ϕ, one can derive first approximations for these parameters. For example, the case where $Pr \sim 1$ results in

$$a_0 = \frac{\frac{210m}{37} + 2}{1 - \frac{\left[\epsilon\ 2Pr^{1/2} - \frac{v_w}{U_e}\sqrt{\frac{4(315)(\rho U_e x)}{37\mu_w}}\right]}{6}} \ . \tag{21}$$

$$b_0 = \frac{2}{1 + \left[\frac{\frac{v_w}{U_e}\sqrt{\frac{4(315)(\rho U_e x)}{37\mu_w}}}{6} Pr^{1/2}\right]} \ . \tag{22}$$

The above equations are the first approximation of the parameters a and b. The procedure for calculating δ, Δ, a, b, λ, and H for arbitrary body shape and arbitrarily distributed but modest magnitude of suction is to calculate a and b from Eqs. (21) and (22). The parameter

m is evaluated locally by using the definition that $m = \left(\frac{dU_e}{dx}\right)\frac{x}{U_e}$. The resulting values of a and b are used in Eqs. (18) and (19) to determine δ and Δ. These results are then used in Eqs. (10)-(15) to calculate approximate values of the boundary-layer parameters.

STABILITY ANALYSIS

There are two possibilities of approximately relating the critical Reynolds number to a boundary-layer parameter. The first is the Schlichting-Ulrich correlation,[3]

$$R_e = 645 \exp(0.6a) \ . \tag{23}$$

The second is the Lin approximation,[4]

$$\pi u'_w \left(\frac{uu''}{u'^3}\right)_c = 0.58 \tag{24}$$

$$\alpha = u'_w c \tag{25}$$

$$R_c c^4 = 25u'_w \ , \tag{26}$$

where the velocity has been normalized with U_e, the subscript prime denotes $d/(d\,\frac{y}{\delta})$, and the Reynolds number is based on displacement thickness, free-stream velocity, and free-stream viscosity.

The Schlichting-Ulrich approximation is the simplest, and it will be the first to be considered. However, a relationship between a and H must be determined because the parameter most used in stability correlations is the shape factor H. From Eqs. (13) and (14) one can show that the parameter H is related to a in the following way:

$$2a = \frac{3}{5H}(4H + 21) - \sqrt{\left[\frac{3}{5H}(4H + 21)\right]^2 + \frac{288}{5H}(2H - 7)} \ . \tag{27}$$

Note that the critical Reynolds number given by Eq. (23) depends only on the shape factor H. Therefore, a universal curve for R_c as a function of H can be calculated that includes the effects of pressure gradient, variable viscosity, and suction.

Let us investigate the Lin approximation to see if the observation holds true there. Using the profile given by Eq. (8), one can show that Eqs. (24)-(26) reduce to

$$-\pi a \left(\frac{ff''}{f'^3}\right)_c = 0.58 \tag{28}$$

$$\alpha = ac \; , \tag{29}$$

$$R_c = \frac{25a}{c^4} \; , \tag{30}$$

where f is defined by Eq. (8). Equation (28) could be used to determine the disturbance wave speed c. This equation states that the critical layer is determined only by the profile parameter a, which implies that c is only a function of the shape factor H. Equation (29) implies that the critical α wave number is a function of H, and Eq. (30) implies that the critical Reynolds number is also a function of H alone.

Two different approximations of the critical parameters for boundary-layer stability have been used to show that the critical parameters are a function of only the shape factor H. This conclusion is independent of the local similarity assumption of the previous section. The implication for correlating experimental and design data is encouraging. For example, Fig. 1 (taken from Ref. 4) shows a correlation of R_c for various pressure gradients and suction effects with the shape factor H. This correlation and Eq. (23) are in fair agreement when $2.3 \le H \le 2.70$.

There are indications that H is not the only possible universal parameter. A parameter discussed and defined as $\bar{\bar{u}}''(0) = u_{yy}(0)/U_e(\delta^2)$

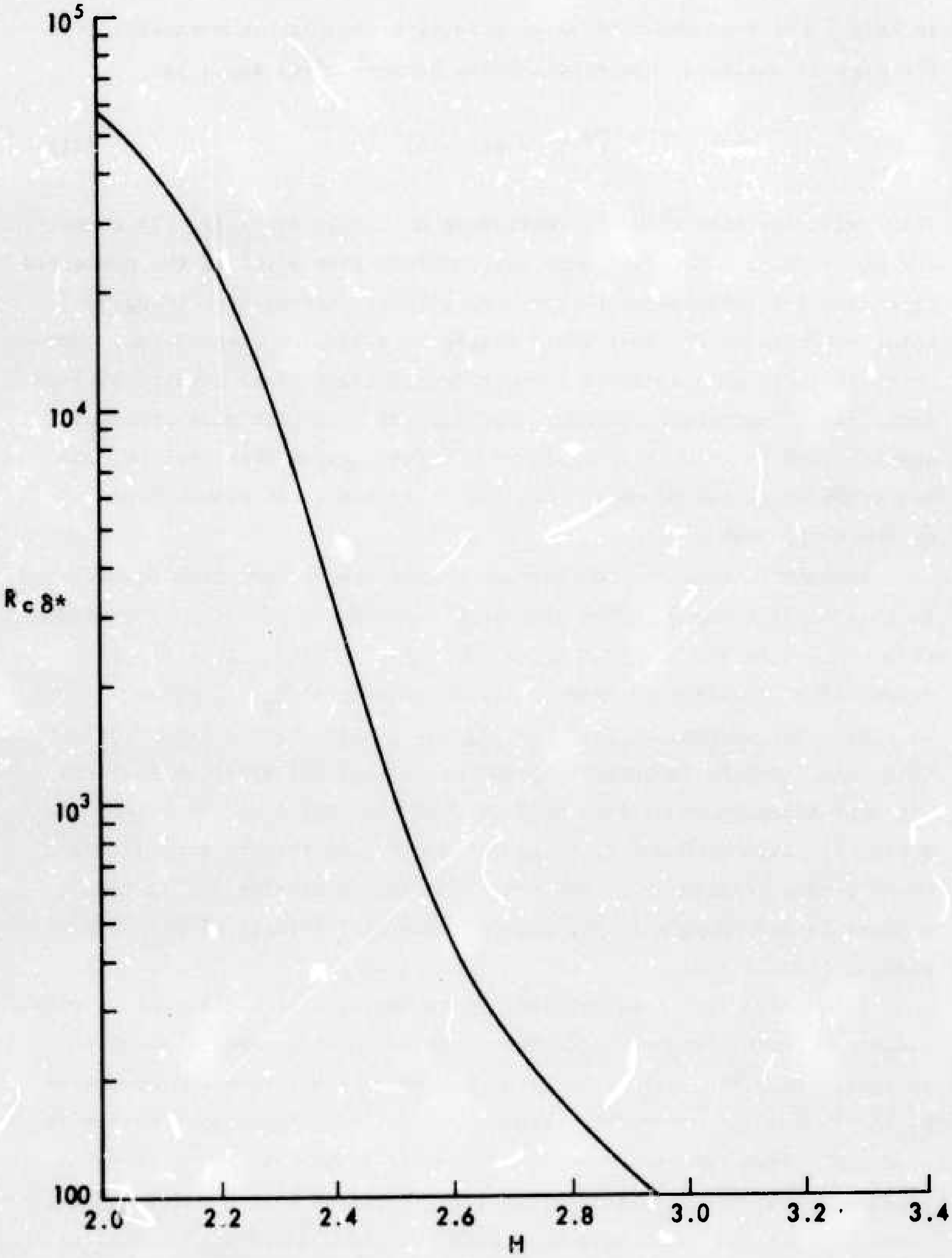

Fig. 1—The critical Reynolds number as a function of the shape parameter

in Ref. 2 has been shown to be an effective correlation parameter. In the present analysis, the relationship between $\bar{\bar{u}}''(0)$ and a is

$$\bar{\bar{u}}''(0) = 6(2 - a) \; . \qquad (31)$$

This indicates that $\bar{\bar{u}}''(0)$ is related to H through Eq. (27). In comparing Eq. (27) with Eq. (31), one may conclude that $\bar{\bar{u}}''(0)$ is the preferred parameter for correlating theoretical results derived from integral solutions because Eq. (31) would result in a simpler computation. However, there is no preference in correlating theoretical results derived from "exact" numerical computer solutions because both parameters are equally easy to calculate. For correlating experimental results, one may prefer H as the parameter because it is easier to deduce from measurements than $\bar{\bar{u}}''(0)$.

Another interesting consideration that can be seen from Eq. (23) is that small changes in the profile parameter, a, can produce considerable changes in the critical Reynolds number. Changes in a are exponentially amplified to produce larger changes in R_c. Further, it can be shown that changes in wall heating and suction have a larger effect than small changes in pressure gradient. It is not apparent that the Lin approximation would substantiate this conclusion to the extent of making it cogent. However, it can be shown that the Lin approximation would produce similar results, small changes in heating and suction produce larger changes in R_c, and the numerical results of Ref. 2 have similar indications.

The reason for these conclusions is that the derivative of R_c with respect to either the wall-heating parameter or the suction parameter is larger than the derivative with respect to the Pohlhausen parameter by the factor a, the profile parameter. The additional implication is that the larger the parameter a or the more favorable the pressure gradient, the more stabilizing are the effects of wall heating and suction. This trend is in agreement with one established with numerical results in Ref. 5.

It is interesting to compare the relative effects of heating and suction. The criterion for the comparison is how large must the physical

boundary conditions, T_w and $\rho v_w/\rho U_e$, be so that both ϕ and Q in Eq. (12) have an order of unity magnitude. According to Refs. 6 and 7, surface conditions of this magnitude have large effects on the boundary-layer flow. For an order unity ϕ and T_e = 60°F, the surface has to be heated to at least 160°F. For an order unity Q and $\rho U_e x/U_w \sim 10^6$, the suction parameter $\rho v_w/\rho U_e$ has to be $\rho v_w/\rho U_e \sim 10^{-3}$. Thus suction appears to be more effective than heating in stabilizing the boundary layer. The implication here is that it is much more difficult to provide a system that will produce wall temperatures on the order of 160°F than to provide a suction velocity that is a tenth of a percent of the free-stream velocity.

DESIGN IMPLICATIONS

For an arbitrary body shape, the results presented here can be used to indicate the critical boundary-layer stability parameters. For example, the Pohlhausen parameter λ can be calculated using the procedure discussed in a previous section. Using results similar to that given in Fig. 2, one can determine the profile parameter a for all heating, $\phi \neq 0$, and arbitrary suction $Q \neq 0$. The value of a is used in Fig. 3, which is a plot of Eq. (27), to determine a value of H; this H is used in Fig. 1 to determine a value of R_c. Note that the trends given by the approximate analysis are correct. Increasing wall heating, one will find from Fig. 2 that a increases for a given λ. Figure 3 shows that as a increases H decreases, and Fig. 1 shows that as H decreases R_c increases.

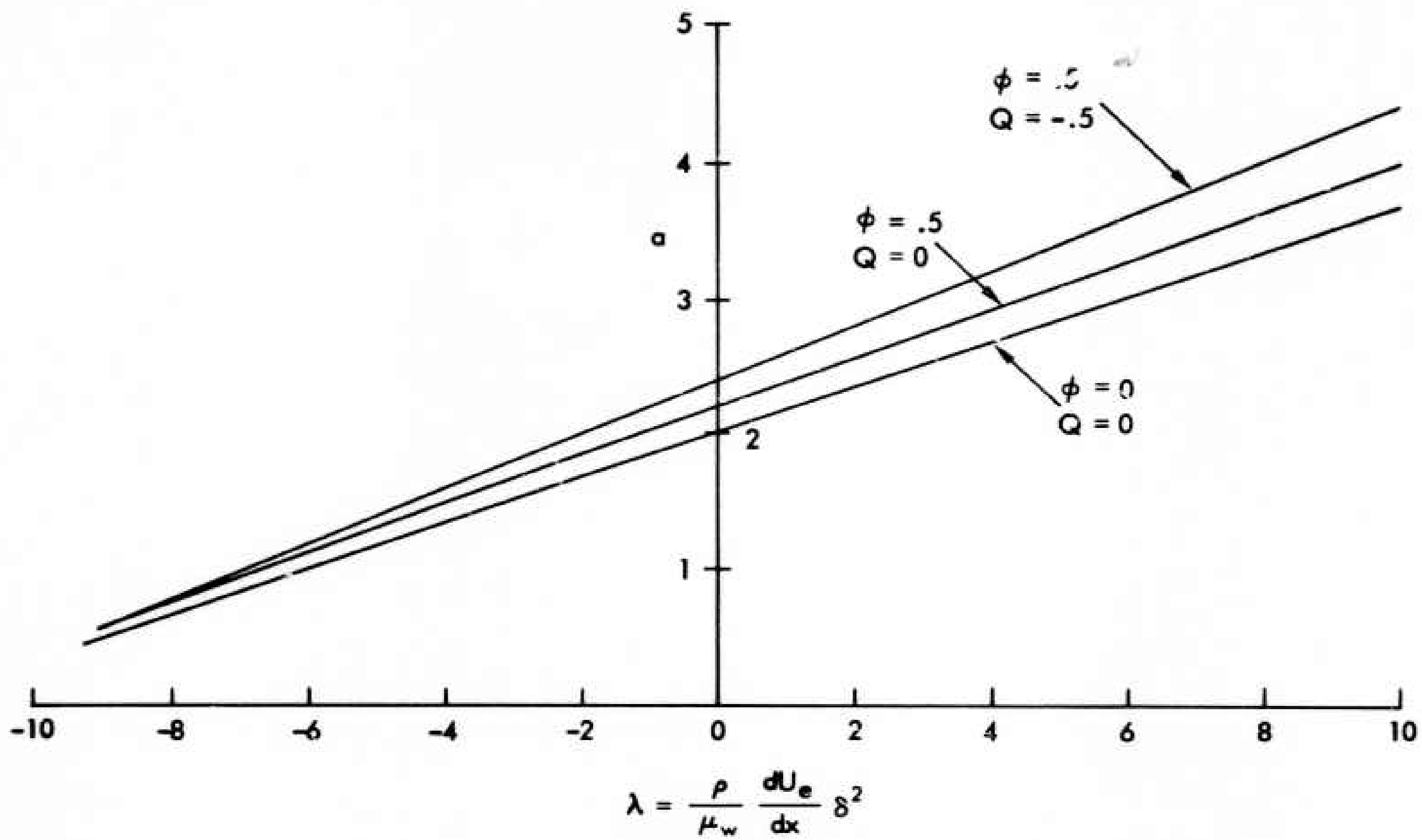

Fig. 2—Profile parameter vs Pohlhausen parameter with suction and heating

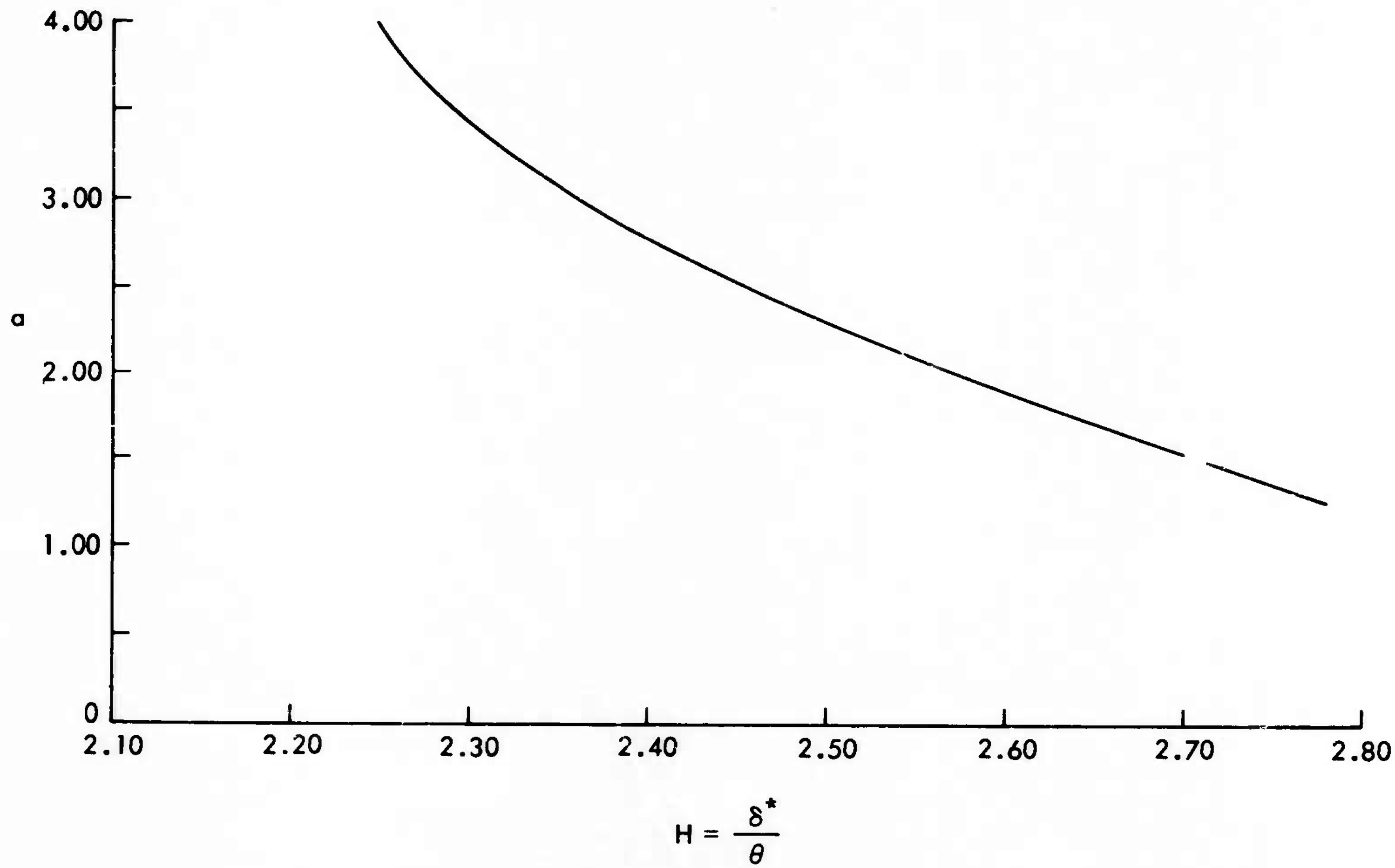

Fig. 3 — Profile parameter vs shape factor

III. CONCLUDING REMARKS

The effects of wall heating and ction on the stability of a two-dimensional, laminar boundary layer have been studied by using an integral analysis of the boundary layer, and the Schlichting-Ulrich and Lin approximations to determine the critical Reynolds number. It was shown that the effects of pressure gradient, wall heating, and suction could be incorporated into a universal curve of the critical Reynolds number as a function of either shape factor H or the parameter proposed by Wazzan, $\bar{\bar{u}}''(0)$. Further, it was indicated that small changes in wall heating and suction produce large changes in the critical Reynolds number and that suction is more effective in stabilizing a boundary-layer flow than wall heating. It was also shown that changes in the critical Reynolds number produced by changes in pressure gradient were smaller than those produced by changes in wall heating and suction. The assumption of local-similarity was used to discuss a method for locating the critical Reynolds number.

Although the parameter crucial to designers is the location of transition and not so much the critical Reynolds number, it is possible that the above result can be useful in correlating transition. A further study of existing data and correlations is required.

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